

不定积分练习

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1 不定积分

1.1 含有 $ax + b$ 的积分

1. $\int \frac{dx}{ax + b}$

2. $\int (ax + b)^\mu dx (\mu \neq -1)$

3. $\int \frac{x}{ax + b} dx$

4. $\int \frac{x^2}{ax + b} dx$

5. $\int \frac{dx}{x(ax + b)}$

6. $\int \frac{dx}{x^2(ax + b)}$

7. $\int \frac{x}{(ax + b)^2} dx$

8. $\int \frac{x^2}{(ax + b)^2} dx$

9. $\int \frac{dx}{x(ax + b)^2}$

1.2 含有 $\sqrt{ax + b}$ 的积分

10. $\int \sqrt{ax + b} dx$

11. $\int x\sqrt{ax + b} dx$

12. $\int x^2\sqrt{ax + b} dx$

13. $\int \frac{x}{\sqrt{ax + b}} dx$

14. $\int \frac{x^2}{\sqrt{ax + b}} dx$

15. $\int \frac{dx}{x\sqrt{ax + b}}$

16. $\int \frac{dx}{x^2\sqrt{ax + b}}$

17. $\int \frac{\sqrt{ax + b}}{x} dx$

18. $\int \frac{\sqrt{ax + b}}{x^2} dx$

1.3 含有 $x^2 \pm a^2$ 的积分

19. $\int \frac{dx}{x^2 + a^2}$

20. $\int \frac{dx}{(x^2 + a^2)^n}$

21. $\int \frac{dx}{x^2 - a^2}$

1.4 含有 $ax^2 + b (a > 0)$ 的积分

22. $\int \frac{dx}{ax^2 + b}$

23. $\int \frac{x}{ax^2 + b} dx$

24. $\int \frac{x^2}{ax^2 + b} dx$

25. $\int \frac{dx}{x(ax^2 + b)}$

26. $\int \frac{dx}{x^2(ax^2 + b)}$

27. $\int \frac{dx}{x^3(ax^2 + b)}$

28. $\int \frac{dx}{(ax^2 + b)^2}$

1.5 含有 $ax^2 + bx + c (a > 0)$ 的积分

29.
$$\int \frac{dx}{ax^2 + bx + c}$$

30.
$$\int \frac{x}{ax^2 + bx + c} dx$$

1.6 含有 $\sqrt{x^2 + a^2} (a > 0)$ 的积分

31.
$$\int \frac{dx}{\sqrt{x^2 + a^2}}$$

32.
$$\int \frac{dx}{\sqrt{(x^2 + a^2)^3}}$$

33.
$$\int \frac{x}{\sqrt{x^2 + a^2}} dx$$

34.
$$\int \frac{x}{\sqrt{(x^2 + a^2)^3}} dx$$

35.
$$\int \frac{x^2}{\sqrt{x^2 + a^2}} dx$$

36.
$$\int \frac{x^2}{\sqrt{(x^2 + a^2)^3}} dx$$

37.
$$\int \frac{dx}{x\sqrt{x^2 + a^2}}$$

38.
$$\int \frac{dx}{x^2\sqrt{x^2 + a^2}}$$

39.
$$\int \sqrt{x^2 + a^2} dx$$

40.
$$\int \sqrt{(x^2 + a^2)^3} dx$$

41.
$$\int x\sqrt{x^2 + a^2} dx$$

42.
$$\int x^2\sqrt{x^2 + a^2} dx$$

43.
$$\int \frac{\sqrt{x^2 + a^2}}{x} dx$$

44.
$$\int \frac{\sqrt{x^2 + a^2}}{x^2} dx$$

1.7 含有 $\sqrt{x^2 - a^2} (a > 0)$ 的积分

45.
$$\int \frac{dx}{\sqrt{x^2 - a^2}}$$

46.
$$\int \frac{dx}{\sqrt{(x^2 - a^2)^3}}$$

47.
$$\int \frac{x}{\sqrt{x^2 - a^2}} dx$$

48.
$$\int \frac{x}{\sqrt{(x^2 - a^2)^3}} dx$$

49.
$$\int \frac{x^2}{\sqrt{x^2 - a^2}} dx$$

50.
$$\int \frac{x^2}{\sqrt{(x^2 - a^2)^3}} dx$$

51.
$$\int \frac{dx}{x\sqrt{x^2 - a^2}}$$

52.
$$\int \frac{dx}{x^2\sqrt{x^2 - a^2}}$$

53.
$$\int \sqrt{x^2 - a^2} dx$$

54.
$$\int \sqrt{(x^2 - a^2)^3} dx$$

55.
$$\int x\sqrt{x^2 - a^2} dx$$

56.
$$\int x^2\sqrt{x^2 - a^2} dx$$

57.
$$\int \frac{\sqrt{x^2 - a^2}}{x} dx$$

58.
$$\int \frac{\sqrt{x^2 - a^2}}{x^2} dx$$

1.8 含有 $\sqrt{a^2 - x^2} (a > 0)$ 的积分

59.
$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$

60.
$$\int \frac{dx}{\sqrt{(a^2 - x^2)^3}}$$

61.
$$\int \frac{x}{\sqrt{a^2 - x^2}} dx$$

62.
$$\int \frac{x}{\sqrt{(a^2 - x^2)^3}} dx$$

63.
$$\int \frac{x^2}{\sqrt{a^2 - x^2}} dx$$

64.
$$\int \frac{x^2}{\sqrt{(a^2 - x^2)^3}} dx$$

65.
$$\int \frac{dx}{x\sqrt{a^2 - x^2}}$$

66. $\int \frac{dx}{x^2\sqrt{a^2-x^2}}$

67. $\int \sqrt{a^2-x^2} dx$

68. $\int \sqrt{(a^2-x^2)^3} dx$

69. $\int x\sqrt{a^2-x^2} dx$

70. $\int x^2\sqrt{a^2-x^2} dx$

71. $\int \frac{\sqrt{a^2-x^2}}{x} dx$

72. $\int \frac{\sqrt{a^2-x^2}}{x^2} dx$

1.9 含有 $\sqrt{\pm ax^2+bx+c}$ ($a > 0$) 的积分

73. $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

74. $\int \sqrt{ax^2+bx+c} dx$

75. $\int \frac{x}{\sqrt{ax^2+bx+c}} dx$

76. $\int \frac{dx}{\sqrt{c+bx-ax^2}}$

77. $\int \sqrt{c+bx-ax^2} dx$

78. $\int \frac{x}{\sqrt{c+bx-ax^2}} dx$

1.10 含有 $\sqrt{\pm \frac{x-a}{x-b}}$ 或 $\sqrt{(x-a)(b-x)}$ 的积分

79. $\int \sqrt{\frac{x-a}{x-b}} dx$

80. $\int \sqrt{\frac{x-a}{b-x}} dx$

81. $\int \frac{dx}{\sqrt{(x-a)(b-x)}}$

82. $\int \sqrt{(x-a)(b-x)} dx$

1.11 含有三角函数的积分

83. $\int \sin x dx$

84. $\int \cos x dx$

85. $\int \tan x dx$

86. $\int \cot x dx$

87. $\int \sec x dx$

88. $\int \csc x dx$

89. $\int \sec^2 x dx$

90. $\int \csc^2 x dx$

91. $\int \sec x \cdot \tan x dx$

92. $\int \csc x \cdot \cot x dx$

93. $\int \sin^2 x dx$

94. $\int \cos^2 x dx$

95. $\int \sin^n x dx$

96. $\int \cos^n x dx$

97. $\int \frac{dx}{\sin^n x}$

98. $\int \frac{dx}{\cos^n x}$

99. $\int \cos^m x \cdot \sin^n x dx$

100. $\int \sin ax \cdot \cos bx dx$

101. $\int \sin ax \cdot \sin bx dx$

102. $\int \cos ax \cdot \cos bx dx$

103. $\int \frac{dx}{a+b \cdot \sin x} (a^2 > b^2)$

$$104. \int \frac{dx}{a + b \cdot \sin x} (a^2 < b^2)$$

$$105. \int \frac{dx}{a + b \cdot \cos x} (a^2 > b^2)$$

$$106. \int \frac{dx}{a + b \cdot \cos x} (a^2 < b^2)$$

$$107. \int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$108. \int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x}$$

$$109. \int x \cdot \sin ax \, dx$$

$$110. \int x^2 \cdot \sin ax \, dx$$

$$111. \int x \cdot \cos ax \, dx$$

$$112. \int x^2 \cdot \cos ax \, dx$$

1.12 含有反三角函数的积分 ($a > 0$)

$$113. \int \arcsin \frac{x}{a} dx$$

$$114. \int x \cdot \arcsin \frac{x}{a} dx$$

$$115. \int x^2 \cdot \arcsin \frac{x}{a} dx$$

$$116. \int \arccos \frac{x}{a} dx$$

$$117. \int x \cdot \arccos \frac{x}{a} dx$$

$$118. \int x^2 \cdot \arccos \frac{x}{a} dx$$

$$119. \int \arctan \frac{x}{a} dx$$

$$120. \int x \cdot \arctan \frac{x}{a} dx$$

$$121. \int x^2 \cdot \arctan \frac{x}{a} dx$$

1.13 含有指数函数的积分

$$122. \int a^x dx$$

$$123. \int e^{ax} dx$$

$$124. \int x \cdot e^{ax} dx$$

$$125. \int x^n \cdot e^{ax} dx$$

$$126. \int x \cdot a^x dx$$

$$127. \int x^n \cdot a^x dx$$

$$128. \int e^{ax} \cdot \sin bx \, dx$$

$$129. \int e^{ax} \cdot \cos bx \, dx$$

$$130. \int e^{ax} \cdot \sin^n bx \, dx$$

$$131. \int e^{ax} \cdot \cos^n bx \, dx$$

1.14 含有对数函数的积分

$$132. \int \ln x \, dx$$

$$133. \int \frac{dx}{x \cdot \ln x}$$

$$134. \int x^n \cdot \ln x \, dx$$

$$135. \int (\ln x)^n dx$$

$$136. \int x^m \cdot (\ln x)^n dx$$

1.15 含有双曲函数的积分

$$137. \int \operatorname{sh} x \, dx$$

$$138. \int \operatorname{ch} x \, dx$$

$$139. \int \operatorname{th} x \, dx$$

$$140. \int \operatorname{sh}^2 x \, dx$$

$$141. \int \operatorname{ch}^2 x \, dx$$

2 定积分

$$142. \int_{-\pi}^{\pi} \cos nx \, dx = \int_{-\pi}^{\pi} \sin nx \, dx = 0$$

$$143. \int_{-\pi}^{\pi} \cos mx \cdot \sin nx \, dx = 0$$

$$144. \int_{-\pi}^{\pi} \cos mx \cdot \cos nx \, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$147. I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx = \int_0^{\frac{\pi}{2}} \cos^n x \, dx$$

$$I_n = \frac{n-1}{n} I_{n-2}$$

$$= \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{4}{5} \cdot \frac{2}{3} (n \text{ 为大于1 的正奇数}), I_1 = 1 \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} (n \text{ 为正偶数}), I_0 = \frac{\pi}{2} \end{cases}$$

$$145. \int_{-\pi}^{\pi} \sin mx \cdot \sin nx \, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$146. \int_0^{\pi} \sin mx \cdot \sin nx \, dx \\ = \int_0^{\pi} \cos mx \cdot \cos nx \, dx \\ = \begin{cases} 0, & m \neq n \\ \frac{\pi}{2}, & m = n \end{cases}$$